# Permutations

1. B 24

2. C 6

3. A 120 ways

4. B 120 ways

5. C 48 words

6. C 6!

7. A 4! \* 3!

8. B 15

# Permutations and Combinations

**1. How many numbers are there between 99 and 1000, having at least one of their digits 7?**

**Solution:**

Numbers between 99 and 1000 are all three-digit numbers.

Total number of 3 digit numbers having at least one of their digits as 7 = (Total numbers of three-digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all)

= (9 × 10 × 10) – (8 × 9 × 9)

= 900 – 648

= 252

**2. How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?**

**Solution:**

Let ABCDE be a five-digit number.

Given that the first two digits of each number are 6 and 7.

Therefore, the number is 67CDE.

As repetition is not allowed and 6 and 7 are already taken, the digits available for place C are 0, 1, 2, 3, 4, 5, 8, 9, i.e. eight possible digits.

Suppose one of them is taken at C, now the digits possible at place D is 7.

Similarly, at E, the possible digit is 6.

Therefore, the total five-digit numbers with given conditions = 8 × 7 × 6 = 336.

**3. Find the number of permutations of the letters of the word ALLAHABAD.**

**Solution:**

Given word – ALLAHABAD

Here, there are 9 objects (letters) of which there are 4As, 2 Ls and rest are all different.

Therefore, the required number of arrangements = 9!/(4! 2!)

= (1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9)/ (1 × 2 × 3 × 4 × 1 × 2)

= (5 × 6 × 7 × 8 × 9)/2

= 7560

**4. In how many of the distinct permutations of the letters in MISSISSIPPI do the four Is not come together?**

**Solution:**

Given word – MISSISSIPPI

M – 1

I – 4

S – 4

P – 2

Number of permutations = 11!/(4! 4! 2!) = (11 × 10 × 9 × 8 × 7 × 6 × 5 × 4!)/ (4! × 24 × 2)

= 34650

We take that 4 I’s come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = 8!/(4! 2!)

= (8 × 7 × 6 × 5 × 4!)/ (4! × 2)

= 840

Therefore, the total number of permutations where four Is don’t come together = 34650 – 840 = 33810

**5. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?**

**Solution:**

Given,

Total number of families = 87

Number of families with at most 2 children = 52

Remaining families = 87 – 52 = 35

Also, for the rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children.

Thus, the following are the number of possible choices:

52C18 × 35C2 (18 families having at most 2 children and 2 selected from other types of families)

52C19 × 35C1 (19 families having at most 2 children and 1 selected from other types of families)

52C20 (All selected 20 families having at most 2 children)

Hence, the total number of possible choices = 52C18 × 35C2 + 52C19 × 35C1 + 52C20

**6. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?**

**Solution:**

Given,

Men = 2

Women = 3

A committee of 3 persons to be constituted.

Here, the order does not matter.

Therefore, we need to count combinations.

There will be as many committees as combinations of 5 different persons taken 3 at a time.

Hence, the required number of ways = 5C3

= 5!/(3! 2!)

= (5 × 4 × 3!)/(3! × 2)

= 10

Committees with 1 man and 2 women:

1 man can be selected from 2 men in 2C1 ways.

2 women can be selected from 3 women in 3C2 ways.

Therefore, the required number of committees = 2C1 × 3C2

= 2 × 3C1

= 2 × 3

= 6

**7. Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one ace in each combination.**

**Solution:**

Given a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to the given, we need to select 1 Ace card out of the 4 Ace cards

∴ The number of ways to select 1 Ace from 4 Ace cards is 4C1

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ The number of ways to select 4 cards from 48 cards is 48C4

Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination = 4C1 × 48C4

= 4 × [48!/(44! 4!)]

= 4 × [(48 × 47 × 46 × 45 × 44!)/ (44! × 24)]

= 4 × 2 × 47 × 46 × 45

= 778320

**8. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has**

**(i) no girls**

**(ii) at least one boy and one girl**

**(iii) at least three girls**

**Solution:**

Given,

Number of girls = 7

Number of boys = 7

(i) No girls

Total number of ways the team can have no girls = 4C0 × 7C5

= 1 × 21

= 21

(ii) at least one boy and one girl

1 boy and 4 girls = 7C1 × 4C4 = 7 × 1 = 7

2 boys and 3 girls = 7C2 × 4C3 = 21 × 4 = 84

3 boys and 2 girls = 7C3 × 4C2 = 35 × 6 = 210

4 boys and 1 girl = 7C4 × 4C1 = 35 × 4 = 140

Total number of ways the team can have at least one boy and one girl = 7 + 84 + 210 + 140

= 441

(iii) At least three girls

Total number of ways the team can have at least three girls = 4C3 × 7C2 + 4C4 × 7C1

= 4 × 21 + 7

= 84 + 7

= 91

**9. How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4?**

**Solution:**

Given numbers – 1000000

Number of digits = 7

The numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

When 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2s and 2, 4s.

Thus, the number of numbers beginning with 1 = 6!/(3! 2!) = (6 × 5 × 4 × 3!)/(3! × 2)

= 60

The total numbers begin with 2 = 6!/(2! 2!) = 720/4 = 180

Similarly, the total numbers beginning with 4 = 6!/3! = 720/6 = 120

Therefore, the required number of numbers = 60 + 180 + 120 = 360.

**10. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?**

**Solution:**

Number of mice = 18

Number of groups = 3

Since the groups are equally large, the possible number of mice in each group = 18/3 = 6

The number of ways of placement of mice =18!

For each group, the placement of mice = 6!

Hence, the required number of ways = 18!/(6!6!6!) = 18!/(6!)3